THE GALACTIC STAR CATALOG AND TABLES PROPOSED BY INNES.

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CATALOG AND FORMULÆ.

The essential principle for a permanent star catalog must be a system of invariable co-ordinates. The equatorial co-ordinates which have always been used are continually changing because of precession of the equinox. A natural system would be one related to the natural features of the stellar system. Mr. Innes proposes for the fundamental plane the one suggested by the peculiar configuration of the stars, the Milky Way, and for the zero point of longitude, the longitude of the apex of the Sun's motion. He has presented in the South African Union Observatory *Circulars*[1] the reasons for, and advantages of, the galactic system of star places, with the necessary reduction formulæ and tables and examples of their use. Since the reference system for the stars is a matter of essential importance in the practical use of star catalogs and the study of the stellar universe, a concise synopsis of the formulæ, an illustration of the proposed catalog in galactic co-ordinates, and a brief discussion of the theme is here presented. Illustrations of the tables and computations cannot be conveniently included here, but may be found in Mr. Innes's papers.

A star catalog in galactic co-ordinates would contain in successive columns the star's name or designation, magnitude, spectrum, galactic longitude (λ), centennial proper-motion in longitude, galactic latitude (β), centennial proper-motion in latitude, right ascension (α) plus centennial precession, declination (δ) plus centennial precession, and finally the authority for the given co-ordinates. A short catalog is given below for illustration. The values of λ and β are given with full accuracy, but α and δ are given only to tenths of a minute of time for α and to the minute of arc for δ. These approximate α's and δ's are sufficient for use at the telescope and by means of the adjoined values of precession they suffice for identifications at different epochs.

[1] Numbers 2, 5, 6, 9, 10; 1912, 1913.

The formula for the conversion of observed right ascensions and declinations into galactic longitudes and latitudes for the catalog are:—

$$\begin{aligned}
\cos\beta\cos(\lambda+D) &= \cos\delta\cos(\alpha-\Omega)\\
\cos\beta\sin(\lambda+D) &= \cos\delta\sin(\alpha-\Omega)\cos\iota+\sin\delta\sin\iota\\
\sin\beta &= -\cos\delta\sin(\alpha-\Omega)\sin\iota+\sin\delta\cos\iota
\end{aligned}$$

in which

Ω = right ascension of the ascending node of the galactic plane,
ι = inclination of the galactic plane to the equator,
D = angle between node and zero point of longitude.

The zero point of longitude is assumed to be at the meridian of longitude passing thru the solar apex. For finding α and δ from the catalog place, the formulæ are:—

$$\begin{aligned}
\cos\delta\cos(\alpha-\Omega) &= \cos\beta\cos(\lambda+D)\\
\cos\delta\sin(\alpha-\Omega) &= \cos\beta\sin(\lambda+D)\cos\iota-\sin\beta\sin\iota\\
\sin\delta &= \cos\beta\sin(\lambda+D)\sin\iota+\sin\beta\cos\iota
\end{aligned}$$

The solution of these formulæ may be performed by means of auxiliary angles, as shown in text-books on practical and spherical astronomy, or more conveniently and precisely by direct computation with the help of ZECH'S addition and subtraction logarithms. To obtain quickly from λ and β the approximate α and δ for use at the telescope, a table may be formed in which precession factors are given similarly, as shown below in the catalog. It may not be a convenient form for the reverse operation, and a reverse table would be of use for only a few years. However, rapid computation of λ and β from α and δ may be effected with the help of a table in connection with the following formulæ, which connect the co-ordinates α, δ, λ, β of the star with those, α, δ_0,λ_0, $\beta = 0$, of the point of intersection of the star's declination arc with the galactic plane:—

$$\begin{aligned}
\sin\eta_0 &= \cos(\alpha-\Omega)\sin\iota\\
\tan(\lambda_0+D) &= \tan(\alpha-\Omega)\sec\iota\\
\tan\delta_0 &= \sin(\alpha-\Omega)\tan\iota\\
\tan\alpha_2 &= \sin\eta_0\tan(\delta-\delta_0)\\
\tan\eta &= \tan\eta_0\sec(\delta-\delta_0)\\
\tan\beta &= \cos\eta\tan(\delta-\delta_0)\\
\sin\beta &= \cos\eta_0\sin(\delta-\delta_0)\\
\lambda &= \lambda_0+\alpha_2=\alpha+\alpha_1+\alpha_2
\end{aligned}$$

in which

η = parallactic angle at the star with respect to the north pole and galactic pole,
η_0 = parallactic angle at λ_0, $\beta = 0$, with respect to the north pole and galactic pole,
a_2 = arc of longitude between the star's declination arc and latitude arc,
$a_1 = \lambda_0 - a$,

where the

quadrant of a_2 is same as that of $(\delta - \delta_0)$ when η_0 is +,
quadrant of a_2 is same as that of $-(\delta - \delta_0)$ when η_0 is —,
quadrant of η is first or fourth according to sign, + or —,
of tan η_0 sec $(\delta - \delta_0)$.

If now η_0, λ_0, and δ_0 are tabulated with right ascension as argument, then λ and β are found by a very short calculation. Mr. INNES has constructed a 12-hour table giving to four places of logarithms and tenths of a minute of arc, the values of log sin η_0, log tan η_0, δ_0, a_1, for each hour and minute of right ascension for the mean equinox of 1900. If a exceeds 12 hours, the table may be used with $a - 12^h$, finally adding 180° to λ and changing the signs of η and β. The resulting λ and β are found to the nearest minute of arc which is sufficient for many uses of galactic co-ordinates.

The determination of the elements of the galactic plane for any epoch takes the place of the work of reducing equatorial co-ordinates of different epochs to the same epoch, and must precede the conversion of co-ordinates to and from galactic co-ordinates. The elements are ☊, ι, D, which are defined above. Taking as the fundamental galactic plane that determined by NEWCOMB for 1900 and as the place of the solar apex that adopted by CAMPBELL (assumed to be for 1900), we have for the—

Galactic pole, $a = 191°.1$, $\delta = +26°.8$,

and for the

Sun's apex, $a = 270°$, $\delta = +30°$.

Whence

$$\begin{aligned} 1900\,☊ &= 281°\ 6'\ 0''.00 \\ 1900\,\iota &= 63\ 12\ 0\ .00 \\ 1900\,D &= 23\ 35\ 27\ .26 \end{aligned}$$

From NEWCOMB'S value of the precession of the equinox, etc., Ω, ι, D, may be found for the epochs and, by interpolation, for the beginning of any year from the formulæ:—

$$\begin{aligned}
\Omega &= 281^\circ\ 6'\ 0''.00 + 4413''.57\,T - 6''.88\,T^2 + 0''.186\,T^3 \\
\iota &= 63\ 12\ 0\ .00 + 1967\ .18\,T - 4\ .56\,T^2 - 0\ .145\,T^3 \\
D &= 23\ 35\ 27\ .26 + 432\ .40\,T + 22\ .46\,T^2 - 0\ .128\,T^3
\end{aligned}$$

in which T = solar centuries from 1900. Mr. INNES has tabulated these values for five-year intervals from 1750 to 1950 and for every year from 1900 to 1925.

A ten-day ephemeris of Ω, ι, D for any year may be constructed by means of the formulæ:—

$$\begin{aligned}
\Omega_1 &= \Omega + f - g\cos(\mathrm{G}+\Omega)\cot\iota \\
\iota_1 &= \iota \quad - g\sin(\mathrm{G}+\Omega) \\
D_1 &= D \quad + g\cos(\mathrm{G}+\Omega)\operatorname{cosec}\iota
\end{aligned}$$

in which f, g, G are star-numbers given in the Nautical Almanacs. From this ephemeris Ω_1, ι_1, D_1, may be taken for any day of the year and with Ω_1, ι_1, D_1, the apparent place in right ascension and declination may be derived by a single computation. Thus is eliminated the reduction to apparent place which is usual in equatorial co-ordinates. Mr. INNES has constructed an ephemeris for 1913 and 1914, which gives, for greater convenience, the values of log sin ι and log cos ι in place of ι.

A ten-day ephemeris of aberration factors for the aberrational displacements in longitude and latitude may be constructed from the following formulæ:—

$$\begin{aligned}
\Omega_2 &= 268^\circ\ 34'\ 28''.27 + 5051''.49\,T + 1''.15\,T^2 \\
\iota_2 &= 61\ 10\ 56\ .83 + 3\ .84\,T + 0\ .21\,T^2 \\
D_2 &= 50\ 3\ 42\ .04 - 53\ .65\,T - 0\ .01\,T^2 \\
h\sin\mathrm{H}' &= -\mathrm{K}\cos\iota_2\cos(\odot - \Omega_2) \\
h\cos\mathrm{H}' &= -\mathrm{K}\sin(\odot - \Omega_2) \\
-i &= -\mathrm{K}\sin\iota_2\cos(\odot - \Omega_2) \\
\mathrm{H} &= \mathrm{H}' + D_2 \\
\Delta\lambda &= h\sin(\mathrm{H}+\lambda)\sec\beta \\
\Delta\beta &= h\cos(\mathrm{H}+\lambda)\sin\beta + i\cos\beta
\end{aligned}$$

in which

K = constant of aberration
$\odot$ = Sun's longitude
$\Delta\lambda$ = aberrational displacement in longitude
$\Delta\beta$ = aberrational displacement in latitude

while Ω_2, ι_2, D_2 are the elements of the galactic plane with respect to the ecliptic, and h, H, i, are numbers which may be tabulated like BESSEL'S star-numbers. Mr. INNES has named the quantities h, H, log i, Ω, D, log sin ι, log cos ι (omitting the subscript), galactic day-numbers, and gives a table of them for 1913 and 1914.

A ten-day ephemeris of the right ascension and declinations of any star may be constructed by the following differential method, which avoids the repeated seven-figure calculation of conversion for each date. The method requires the right ascension and declination of the star for a single date within the interval of the ephemeris and computed with the corresponding values of λ, β, D, Ω, ι. This is most conveniently taken at the middle point of the interval; for instance, June 30th for a year's ephemeris. The differences $\Delta\alpha$ and $\Delta\delta$ for each tenth day of the year are then computed by the following formulæ:—

$$\Delta\alpha = c\,\mathrm{C} + d\,\mathrm{D} + e\,\mathrm{E} + \mathrm{F}$$
$$\Delta\delta = c'\mathrm{C} + d'\mathrm{D} + e'\mathrm{E}$$

$a = \cos(\lambda + D_2)\sec\beta$	$a' = -\sin(\lambda + D_2)\sin\beta - \tan$ [illegible] $\cos\beta$
$b = \sin(\lambda + D_2)\sec\beta$	$b' = \cos(\lambda + D_2)\sin\beta$
$c = \cos\eta\cos\beta\sec\beta$	$c' = \sin\eta\cos\beta$
$d = -\sin\eta\sec\delta$	$d' = \cos\eta$
$e = -\tan\delta\sec(\alpha - \Omega)$	$e' = \sin(\alpha - \Omega)$
$\mathrm{A} = -[0.9942]\cos(\odot - \Omega_2)$	$\mathrm{C} = \Delta\lambda + \Delta D + \mu_\lambda$
$\mathrm{B} = -[1.3111]\sin(\odot - \Omega_2)$	$\mathrm{D} = \Delta\beta + \mu_\beta$
$\Delta\lambda = a\,\mathrm{A} + b\,\mathrm{B}$	$\mathrm{E} = \Delta\iota$
$\Delta\beta = a'\mathrm{A} + b'\mathrm{B}$	$\mathrm{F} = \Delta\Omega$

in which a, a', b, b', c, c', d, d', e, e' are star-numbers in the galactic system, and log A, log β, ΔD, E, F are day-numbers, which may be tabulated in a ten-day ephemeris and given in the astronomical almanacs. η is the parallactic angle and μ_λ and μ_β are the proper motions, while $\Delta\lambda$ and $\Delta\beta$ are as before the reductions for aberration. Mr. INNES has constructed the ephemeris of day-numbers for 1913 and 1914 and an ephemeris for ϵ *Orionis* for 1913.

CATALOG IN GALACTIC CO-ORDINATES.

Star	Mag.	Spect.	λ	μ_λ	β	μ_β	α 1900 + T	δ 1900 + T	Authority
			° ′ ″	″	° ′ ″	″	h m m	° ′ ″	
31 *Cornæ*	5.1	G2	43 48 2.06	+75.40	+88 36 24.38	+ 3.37	12 46.8 + 4.9	+28 5 — 33	NEWCOMB
Parissima	9.3	..	66 19 15.26	+ 3.26	+26 53 22.87	+ 0.30	15 29.4 —172.6	+89 53 — 31	COURVOISIER
Polaris	2.1	F8	66 38 3.53	+ 4.55	+25 35 26.88	+ 0.80	1 22.6 + 67.2	+88 46 + 30	NEWCOMB
α *Persei*	1.9	F5	90 14 39.72	+ 4.00	— 6 28 6.25	— 0.71	3 17.2 + 7.2	+49 30 + 21	NEWCOMB
κ *Persei*	4.0	K	90 52 23.45	+22.60	—12 1 44.38	— 5.32	3 2.7 + 6.7	+44 29 + 23	Gr. 1900 II
Aldebaran	1.1	K5	121 55 29.89	+20.41	—20 17 52.28	— 6.12	4 30.2 + 6.7	+16 19 + 12	NEWCOMB
ε *Orionis*	1.8	B	149 5 17.46	— 0.11	—16 52 50.86	+ 0.04	5 31.1 + 5.1	— 1 16 + 4	NEWCOMB
9 *Puppis*	5.3	F8	175 46 26.06	+24.92	+ 7 22 5.08	—25.01	7 47.1 + 4.6	—13 38 — 15	Gr. 1900 II
Archernar	0.6	B5	233 58 43.09	—11.79	—57 50 20.05	+ 6.77	1 34.0 + 3.7	—57 45 + 30	NEWCOMB
Brisbane 3574	7.8	..	236 25 40.42	—50.40	+ 0 27 16.41	— 9.30	11 20.3 + 4.4	—61 6 — 33	Circ. No. 6
P. G. C. 4448	5.6	A	317 38 43.07	— 1.98	+11 13 54.68	— 3.29	17 29.2 + 5.6	—11 10 — 4	Boss
P. G. C. 4412	7.8	..	317 56 46.47	— 3.27	+14 28 34.89	+ 2.76	17 18.5 + 5.5	— 9 16 — 6	Boss
μ *Ophinchi*	4.6	B8	320 48 5.86	— 2.45	+12 6 52.19	— 0.85	17 32.4 + 5.4	— 8 3 — 4	Boss
P. G. C. 4423	4.6	F	322 7 53.21	— 5.21	+16 1 22.52	+ 5.61	17 21.3 + 5.3	— 5 0 — 5	Boss
P. G. C. 4442	5.7	A	322 23 46.06	—11.09	+14 12 41.22	— 1.11	17 28.2 + 5.3	— 5 40 — 5	Boss
4 *Aquilæ*	5.0	B5	337 38 14.83	— 1.76	+ 1 54 43.94	— 1.39	18 39.8 + 5.0	+ 1 57 + 6	Boss

The effect of the solar motion is well shown here in the tendency of the proper-motions in longitude to change sign towards the anti-apex.

DISCUSSION.

The natural plane for the stars is the galactic plane, instead of the continually shifting Earth's equator and equinox, whose motions complicate all work and investigation unnecessarily. The work of conversion of co-ordinates Mr. INNES shows to be comparatively simple and not much different from that of reducing a star-place from the beginning of the year to date, while reduction from epoch to epoch is never required in the galactic system.

The advantages of galactic catalogs and co-ordinates are concisely these: Reductions to date require less work than the present system. All catalogs, whatever their epoch, will be comparable, since differences of star-places will be caused only by proper-motion. Proper-motions and facts of stellar distribution and drift appear directly from the catalogs.

Precise equatorial co-ordinates will always be required, first, for about three hundred standard stars or clock and fundamentals stars for observations of Sun, Moon, and planets, etc.; second, for six to twelve thousand reference stars for the astrographic charts, equatorial comparison stars for comets, etc., and latitude variation. The remainder of the seven and a half million star-places which will ultimately be contained in the astrographic catalogs will never be required in equatorial co-ordinates. The unvarying galactic co-ordinates are consequently more proper and advantageous for the general star catalogs.

The adoption of galactic co-ordinates will facilitate the inversion of the present method of determining proper motions. In the comparison of charts of different epochs the original bright stars of reference will not be fundamental, but instead, the totality of faint stars whose proper motions are small. The proper-motions of the brighter stars will be determined from the faint stars and not from other bright stars. The work of BURNHAM, BARNARD and others indicates in general the almost absolute fixity of the very faint stars.

Permanent photographic regions are not possible when defined with respect to the equator and equinox. But with the introduction of the galactic system the sky may be divided into about eight hundred overlapping areas of about sixty

square degrees each, whose centers at different epochs will remain the same with respect to the stars and co-ordinates. The same stars will be found on corresponding plates of all epochs. These galactic areas might finally replace the old constellations.

A permanent or uniform numbering system for the stars is not possible for the present astrographic areas and catalogs whose numbers follow different systems. A star cannot be effectively defined by its astrographic number. With the introduction of permanent galactic regions and by limiting designations to generally useful stars, seven to nine stars conspicuous in each square degree could be selected for designation. In this way nearly a half million stars practically uniformly distributed over the sky would receive permanent numbers. The remaining seven million cataloged stars need not be further defined than by their galactic co-ordinates. An example of the designation of a star would be 325, 8, 8, signifying star No. 8 in the square of galactic longitude 325° and latitude 8°. North and south latitudes could be distinguished by italics.

In the application of galactic co-ordinates to the astrographic catalog the formulæ are changed only by writing λ and β for α and δ and L and B as the galactic co-ordinates of the center of the plate. The computation of the equatorial place of a star from an astrographic catalog in galactic standard co-ordinates would require the computation of λ and β by the above-mentioned formulæ and then the conversion of λ and β to α and δ for the given date. The computation of precession from the epoch of the present astrographic catalogs to the beginning of the given year would not enter the process in galactic co-ordinates. Galactic standard co-ordinates would thus offer great advantage for the astrographic catalogs. If the astrographic chart and catalog is repeated in the future on the equatorial system the comparison of the catalogs will be an enormous piece of work and expense. It is to be hoped that the future chart and catalog will be planned on the basis and system outlined by Mr. INNES. The real value of the present astrographic work will lie not in its catalogs, but in its charts, which may be compared with future charts by superposition.

The use of galactic co-ordinates for planetary theories would result in a great saving of work, principally the reductions to an adopted equinox and the conversions to ecliptic longitude and latitude. The orbit constants would remain constant for centuries, changing only because of perturbations.

The objection to galactic co-ordinates that they cannot be of use for observing purposes at the telescope or for immediate identifications is offset by including in the galactic catalog the approximate right ascension and declination with the annual precession. The indefiniteness of the galactic circle and uncertainty of the Sun's apex have been mentioned as difficulties in establishing the galactic system. The positions of the galactic plane and departure point of longitude would be somewhat arbitrary and it might be difficult for astronomers to agree on these points. But future determinations of the plane and apex will differ but little from those that may now be adopted and will cause no inconvenience. Attention has been called to the necessity of continued work with the meridian instruments on the standard stars and the unnecessary labor of conversion to galactic co-ordinates. But the standard star catalog and annual ephemerides would still be constructed in equatorial co-ordinates, as pointed out above, for the few thousand standard and reference stars required. The transition from the equatorial system to the galactic system and the difference between the present and the future catalogs would of course cause difficulties, but these and others would probably not be paramount to the ultimate advantages of the galactic system. The impression that the use of galactic co-ordinates would lead to formulæ of greater complexity than those given by BESSEL is herewith removed by Mr. INNES's presentation of the application of the system. The formulæ and computations are in general shorter. This constitutes a considerable gain, "but the real gain is beyond this. Star-places in galactic co-ordinates are final and comparisons between catalogs become immediate. The deduction of a proper motion to-day requires hours or days of work because of the fictitious movements of the co-ordinate system. With galactic co-ordinates the comparison will be the work of minutes, while yielding proper-motion referred to its natural plane."